DR. DEL'S

# GOLDEN RULE MATH

FOR

## STEM Math Students

21st Century
Math Education
for 21st Century Math Students

Craig Hane, Ph.D.

*Dr. Del's*

# *Golden Rule Math*

**for**

# STEM Math Students

**21st Century Math Education**
**for 21st Century Math Students**

Delbert Craig Hane, Ph.D. (Math) aka Dr. Del
Founder and CEO, Triad Math, Inc.

Published by Triad Math Inc., 3822 S Westmont Avenue,
Bloomington, IN
www.triadmathinc.com
Editor: Debbie Goodman
Interior Design: Stephanie Worden

# Table of Contents

# Introduction

**Math was a Great Barrier to STEM students in the 20th Century.**

In 2009 that Great Barrier was removed!

An amazing Math Tool, miraculous to me, was unleashed on the STEM world.

The result of this 21st Century Math Tool?

**Math is no longer a Great Barrier to STEM students!**

By STEM Math I mean Algebra, Geometry, Analytical Geometry, Linear Algebra, Trigonometry, Calculus, Differential Equations and more.

Today a student can learn all of this Math, and more, by age 16 or 17 if the student starts by age 11 or 12.

Hard to Believe?

Yes, but True!

This is more Revolutionary than the Scientific Calculator was in the 1970's and Personal Computers in the 1980's and 1990's and the Internet in the 1990's the early 2,000's.

This book will explain this and how a STEM student should now be taught Math.

Not only is the Great Math Barrier removed, this Modern 21st Century Tool makes STEM Math much more powerful.

In a Nutshell, this great 21st Century Math Tool is *Wolfram Alpha*.

Www.WolframAlpha.com

And, it's FREE.

The result of this is that STEM Math Education is totally different now than it was prior to 2009. Vastly Superior!

Yet, most of the Math Programs being taught today are still the old 20th Century Programs I call the Standard Math Curriculum.

You will learn all about this in this book.

Furthermore, you will learn about a new SupraComputer which also revolutionizes how you learn and do Math.

The SupraComputer has capabilities no ordinary Personal Computer, Apple or PC has. Why?

The SupraComputer contains Wolfram Alpha whose results can be saved in a Notebook, and also an amazing new programming language, Wolfram Language, released in 2016 which is a modern version of Mathematica, the 1988 computer programming language, which empowered Tim Berners-Lee to create the World Wide Web and Steve Jobs to create Siri for the I-phone.

Finally, you will learn how a revolutionary new Math Program is available Online that empowers any Parent or Student to learn Math very quickly and easily, and also very inexpensively.

**Homeschool Math, for any parent or student, can now be Vastly Superior to Public School Math.**

You will also be directed to many Free Resources, and

You will be given some Special Offers that may be hard to believe.

Math is Fun and Easy to learn and apply in the 21st Century.

Read on to learn How and Why this is.

Truely amazing.

Enjoy!

# Chapter 1: STEM Math in a Nutshell

Science and Engineering are based on Math Models.

Math Models consist of what are called Functions.

There are the standard functions a student learns in PreCalculus like Polynomials, Trigonometric, Exponential functions and their inverse functions, and their composite functions.

There are also what are called Special Functions which are the Solutions of Differential Equations and do not have solutions that are standard functions.

A Differential Equation is often used to create a Math Model for some physical situation or process whose "solution" is a Function.

Sometimes the solution is a standard function learned in PreCalculus, but often it is what is called a Special Function, which is an Infinite Series.

A Function is usually analyzed by Graphing the Function.

Then we understand the behavior of the Function by analyzing its Graph for Max and Min, Inflection Points, Increasing and Decreasing intervals, Concavity, and the Area under its Graph from point a to point b in its domain.

Calculus is for analyzing a Function's behavior.

Differential Equations are for creating the Functions for a certain Math Model.

Calculus and Differential Equations are intimately related and must be understood to understand STEM subjects.

Historically, this was done with wonderful Manual Tools created by our ancestors in the 1600's and 1700's.

One catch!

These Manual Tools are very difficult to learn, master and apply.

And, often they don't work very well.

Integral Calculus is the first application of Differential Equations, called Techniques of Integration, to then apply what is called the Fundamental Theorem of Calculus, FTC.

One must find what is called an Antiderivative of a Function also called the Indefinite Integral of the Function.

This is called Techniques of Integration and there are several "techniques" that are difficult to learn and master.

Then you don't know which Technique will work, and often none of them work!

This is called Integral Calculus, usually Calculus 2.

Calculus 2 probably flunks more potential STEM students than any other course, and is often used as a "filter" by STEM schools.

Just ask any STEM pro or medical doctor you know about their experience and their classmates' experience with Calculus 2.

The Fundamental Theorem of Calculus, FTC, is the most important theorem in mathematics from a STEM perspective.

It was discovered in the 1600's by guys like Newton and Leibniz, and the FTC is the reason modern Technology and Science exists.

Mastering Calculus and Differential Equations has been a great Barrier for a student to learn any STEM subject.

Why?

The Manual Tools are so difficult to learn and master and apply.

Concepts Easy, Manual Calculus Tools HARD!

As you probably know, the Scientific Calculator made arithmetic calculations very easy starting in 1972.

No more Log and Trig Tables, Slide Rules, and Manual arithmetic calculations.

But, Calculus and Differential Equations were still very difficult with the classical Manual Tools.

Now, in the 21st Century, this has all changed thanks to an amazing Math Tool which makes Calculus and Differential Equations very easy too!

**Wolfram Alpha was introduced to the world in 2009.**

Calculus and Differential Equations now became very easy to learn and use.

**The Manual Tools Barrier is gone!**

Sadly, the Math Education community has not caught on yet.

The old Manual Tools are still being taught and this is the Great Barrier for STEM students.

As a Parent, you must be sure your child is taught PreCalculus subjects like Algebra, Analytical Geometry, Trigonometry, Calculus and Differential Equations with this new miracle 21st Century Math Tool, Wolfram Alpha.

Even if your student is subjected to an obsolete 20th Century Math Program, this will help them.

But, ideally they won't be subjected to an obsolete difficult Program.

For a much fuller understanding of this, read Chapter 10.

Then be sure to look at the Special Offers.

You have Nothing to Lose, and Much to Gain!

# Chapter 2: Great Start - Tier 1

A Post-elementary Math student should understand the Decimal Number System.

Then the student is ready for my Six Tier Math Program.

Tier 1 teaches the student how to perform all arithmetic calculations with a TI-30Xa Scientific Calculator.

No more manual, difficult, error prone and time consuming arithmetic calculations. All in just 16 Lessons!

I also review the Rules of Arithmetic in what we call Pre-Algebra to be sure the student understands all the basic rules of arithmetic. Just 10 Lessons!

You may view the Syllabus by going to my website: www.CraigHane.com and downloading the entire Six Tier Syllabus PDF from my home page.

I apply SPIKE Pedagogy you may learn about in Chapter 8.

In essence, the student is given Notes and Exercises in a PDF you may print out or you may buy them in book form at Amazon. These are NOT for self-study.

Each Lesson has a Tutorial Video to explain the Topic, and the Notes are to help the student understand the Tutorial Video and have a written record to review.

The Exercises are where the student really learns, understands and masters the Concepts. Math is like a Sport. Practice, make mistakes, and learn!

The student takes a Pre-test to let the student know what is coming and then a Post-test to be sure the student has mastered and understands the lesson.

This is very important.

Learning Math is like climbing a ladder, Topic by Topic, one step at a time.

Triad Math's Learning Management System, LMS, keeps track of it so the student and his parent or coach can follow the student's progress.

I recommend the student go back and review the Lessons by doing some of the previous lesson's exercises until the student can retain the knowledge.

For some students, like me, this takes many review sessions.

Indeed, if a student can explain a Lesson to someone else that is great too.

Confidence, Self-esteem, and Elimination of Fear are achieved as well.

This is very important.

Most students will go through Tier 1 in a few days or weeks depending on how hard they work.

I recommend short study sessions frequently.

Math is rarely learned well by cramming.

See the SPIKE Pedagogy Chapter 8.

Now your student should be ready to go forward and learn some Math!

Hopefully, your student will begin to like Math like a Sport and Intrinsic Motivation will begin to kick in. See Chapter 12.

The student will now be able to perform arithmetic calculations many adults struggle with. Great for their self-esteem!

One thing students really like at this time is the ability to handle Fractions quickly and easily, including decimal conversions.

The TI-30Xa is a wonderful calculator.

Mastering it for arithmetic calculations empowers a student to now learn more Math much easier without being burdened by the old error prone, time consuming, manual arithmetic calculations.

See the Special Offers to get your student started.

You have Nothing to Lose, and Much to Gain!

# Chapter 3: Great Foundation - Tier 2

What is a great Foundation for any Math Student?

Tier 2

Practical Algebra - 10 Lessons.

Practical Geometry – 19 Lessons.

Practical Trigonometry – 7 Lessons.

What?

Believe it or not!

Have you ever heard of an Algebra 1 course with only 10 Lessons?

This is most of the Practical Algebra one ever needs in a technical subject unless one is going into Science or Engineering.

It's all you need for the Technology component of STEM.

Ditto for Geometry. Only 19 Lessons?

Well, first we need to learn: How long is something?; What is the Area of something"; What is the Volume of something?

That's Practical Geometry.

Go look at the Syllabus.

Remember this is the Foundation.

Why Trigonometry?

Well, Algebra and Geometry are wonderful working together to solve many technical math problems. Even angles of 30, 45, 60, 90, and 180 degrees.

But, they can't deal with arbitrary angles.

Practical Trigonometry does.

Only seven Lessons needed.

A student will be able to deal with any angle problem when done with those lessons.

The TI-30Xa makes this very easy too.

Now, when Tier 2 is mastered, your student will know more Math than most US citizens, including some who are pretty good with technical subjects.

And, the student's Confidence will be quite high, as it should be.

Most students can complete Tier 2 in about 60 hours +/- 30 hours based on their aptitude and efforts.

Confidence and self-esteem should be quite high now.

Tier 2 follows the SPIKE Pedagogy just like all the Tiers do.

PDF Notes and Exercises are provided, or a printed book of them from Amazon is available.

All of the student's progress is recorded in our Learning Management System for the student and parent/coach.

If the student is not college-bound, the student is now ready for the military or an apprentice program in any technical subject.

But, a STEM student usually will go to college.

So on to Tier 3 in Chapter 4.

# Chapter 4: College-Bound - Tier 3

Okay, now to get ready for the SAT or similar tests some colleges may require.

Also, your student will learn some more topics any STEM student will need to know.

Just go look at the Syllabus for Tier 3 PDF at: www.CraigHane.com

Now in addition to Notes, we also begin to use a wonderful book by Dr. George Simmons, *PreCalculus Mathematics in a Nutshell*, that is available on Amazon for about $18 or less.

Wow. A textbook?

Well, no school I know of ever used it as a textbook.

It is 119 pages and very few high school graduates know a lot of what is in this book.

In Tier 3 we use a little of it.

We also teach some “tricky things” a student needs to know for the SAT.

Also, some tips on how to prepare and take the test.

Unfortunately, the SAT is just a tricky test designed to create a Bell Curve distribution and the students who score the highest are often those who got the best SAT prep courses, often from private companies.

Okay, the student scores a perfect SAT or ACT score.

Are they ready for a good STEM school or subject?

You'd think so, wouldn't you?

NO!

Not even close.

Go now to Chapter 5 to see the next Ladder.

# Chapter 5: PreCalculus Math – Tier 4

Algebra, Geometry, Trigonometry and Complex Numbers are the PreCalculus subjects.

Now, a STEM student must learn these subjects at a much deeper level.

Historically, this was very difficult because of the Manual Tools.

Wolfram Alpha is the wonderful 21st Century Tool (2009) that transforms that.

In Tier 4, I introduce Wolfram Alpha in lesson 5 in Algebra, and use it from then on.

I use Dr. Simmons *PreCalculus Mathematics in a Nutshell* when we can, and provide Custom Notes otherwise.

Of course, the Simmons book does not use Wolfram Alpha since it was published in 1988, two decades before Wolfram Alpha appeared.

But, I use Wolfram Alpha along with Dr. Simmons' book.

He explains the Concepts very well.

No other math textbook does this nearly as well so far as I know.

Actually, a STEM student can now go back and review Tier 3 topics and apply Wolfram Alpha there too.

Unfortunately, you can't use Wolfram Alpha on the SAT or ACT, which is why I don't introduce it in Tier 3.

Now the student has a taste of how wonderful it is compared to the old manual techniques.

Complex Numbers are necessary for STEM. They are very easy to understand when taught properly with Euler's Equation and Trigonometry.

So, that is included in Tier 4 too.

Go study the Syllabus for Tier 4 and compare it to any math textbook you might be using or interested in.

Be sure to look at the Special Offers.

# Chapter 6: Calculus, Differential and Integral - Tier 5

If you don't understand or know Calculus, go to Chapter 10 for more details.

If you do know Calculus, then you know there is Differential Calculus, or Calculus 1, which empowers you to analyze Functions and their Graphs. Max, Min, Inflection Points, Concavity, etc.

It uses the concept of Derivative of a Function, i.e. rate of change.

It is pretty easy to find the Derivative of a given function using the Manual Tools like the Chain Rule and the Leibniz Rule.

It is often very difficult to then find the Zeros or Roots of the Derivative Function which you must do to analyze the Function.

Indeed, many times it is impossible or impractical to solve a problem with Manual Tools.

Wolfram Alpha solves all such problems in about a minute, and, indeed, does things no one does manually that help you understand the function.

In Integral Calculus, you find the area under the graph of a function from points a and b in the domain by using the Fundamental Theorem of Calculus, FTC.

This is easy to do IF you can find what is called the Antiderivative or Indefinite Integral of the given function. Then, all you have to do is evaluate the Antiderivative at the points a and b and subtract them. Voila!

The FTC was discovered in the 1600's by Newton in England and Leibniz in Germany. Both Math and Science were revolutionized going forward.

Newton used it to derive Kepler's Planetary Laws of Motion from the simple Gravitational Principle. This was published in his *Principia Mathematica* in about 1687. It launched modern science and technology.

Leibniz actually did things differently using Infinitesimals and his approach to Calculus was conceptually easier, and better, than Newton's.

Leibniz's math students ending with Euler then developed Calculus the way we understand and use it today.

So what's the Problem?

Finding the Antiderivative of a Function can be very difficult and sometimes, indeed often in STEM, impossible in terms of any known standard functions.

This is called Techniques of Integration and probably flunks more students out of a STEM career than any other thing.

Just ask any STEM pro you know about their experience with Calculus 2.

Well, guess what?

Wolfram Alpha finds the Antiderivative for any function, quick, easy and free!

Indeed, if the Antiderivative does not exist in terms of the well-known functions, Wolfram Alpha will find it in terms of a Special Function or an Infinite Series.

This is so difficult it is not taught in most Calculus courses.

Yet, this is what you must do for STEM.

Integral Calculus goes from an 8 in difficulty on a scale of 1 to 10 down to a 1 or 2.

And, you can learn the concepts much better now by doing lots and lots of examples that would be impractical to do manually.

The Calculus 2 Barrier is now removed!

Manual Integration is like a Horse and Buggy.

Wolfram Alpha Integration is like an automobile.

Which would you want to learn?

Do you have to become an expert in Horse and Buggy technology to then learn to drive a modern automobile?

I think not!

In Tier 5, I teach the student how to drive an automobile.

Oops, I mean how to use Wolfram Alpha to solve Calculus problems.

To get started, just go to the Special Offers below.

# Chapter 7: Differential Equations - Tier 6

A Differential Equation has a Function as a Solution.

A Differential Equation is used to create a Math Model for some physical process, i.e. in all STEM subjects.

Solving a Differential Equation is even more difficult than finding Antiderivatives in Integral Calculus.

Indeed, an Antiderivative is the solution of the simplest of all Differential Equations.

So, most students who survive Calculus 2 can make it through Differential Equations.

Just more effort and struggle.

Until now, in the 21st Century!

Believe it or not, Wolfram Alpha solves any Differential Equation quick and easy, and often better than can be done manually. Free too!

Even Differential Equations that are impossible to solve manually in any reasonable time, like the ones whose solutions are Special Functions, which essentially are Infinite Series.

I teach a STEM student how to use Wolfram Alpha to solve Differential Equations in Tier 6.

Now, in about one semester, a STEM student can learn how a Science or Engineer can solve a Differential Equation, quick and easy.

Sort of like going from a Horse and Buggy to an Airplane for long distance travel.

# Chapter 8: SPIKE Pedagogy

Pedagogy means: "The method of teaching a subject."

SPIKE Pedagogy is wonderful for delivering an optimal math education to any student, if you can do it.

Any good Math Tutor knows this.

SPIKE Pedagogy is virtually impossible to practice in a group environment.

Fortunately, Parents can easily practice SPIKE Pedagogy for each of their children thanks to a modern 21st Century Math Program.

Any experienced math teacher or tutor will tell you there are five ingredients of good pedagogy for Math expressed by the acronym SPIKE.

**So, What is SPIKE Pedagogy? S P I K E**

**Self-pacing**. Each student will learn math at his or her own pace which is determined by many factors.

It is difficult, usually impossible, for a student to have self-pacing in a group environment where the Math is being taught on a schedule.

Any good math tutor realizes this, and this is one reason why rich parents use good tutors to teach their children math, especially when they are struggling with math taught in a classroom to a group of students.

Indeed, that is how I made a good living during my school years from ages 15 – 27, high school through graduate school, and learned the value of SPIKE Pedagogy.

**Proper Content**. A student should be taught Math in a sequence of topics so that the student always has the necessary prerequisite math knowledge for each new topic.

Furthermore, it is desirable that the Math topics chosen are of interest and relevance to the student.

This is a horrible failure of our current Standard Math Curriculum taught in most Math Programs which include many obsolete manual tools and much premature theory.

An essential ingredient of proper motivation is to explain to the student how a given topic will serve the student well in the future given the student's larger potential interests in life.

Just realize that Math is a HUGE subject.

No single Human understands or knows all of the Math there is.

Different people need different math topics depending on their broader interests in life.

**Interactivity**. Math is like a sport or game.

To learn math You must do math.

You must practice.

You will make a lot of mistakes.

You will struggle to overcome hurdles.

Fortunately, with the right attitude, Math can be one of the most Fun and Rewarding sports or games you can ever play.

This is why the psychology of the student is so important.

The student must enjoy the sport or game of Math.

This is why having both a great Teacher and a great Coach is so important.

The Teacher explains the Math and selects the proper topics for the student.

The Coach guides and encourages the student.

Mistakes are celebrated as evidence of effort, just like in a sport.

Personal achievements are celebrated as the student climbs the ladder of Math topics.

The Coach must be sure the student doesn't miss any rungs of the ladder.

The Coach must be sure the student practices.

## **Keeping Score**.

It is very important to keep score of a student's progress and recognize the student's progress and achievements, just like any game or sport.

This is an important responsibility of the Coach.

Keeping Score is a powerful motivator for a student.

It is like ranks and merit badges in scouting.

Or, playing a game with one's self.

Climbing the Ladder of Success.

It is important to create and maintain a good psychology for the student.

Math can sometimes be frustrating for virtually any student.

I have a Ph.D. in Math, but I can tell you that many, many times I was frustrated.

I probably have made more mistakes in Math than anyone you know.

Remember, Babe Ruth was the Strikeout King, as well as the Home Run King.

Proper Math Content will vastly improve most students' lives.

Keeping score will prove this when you compare it to the student's other achievements, especially those that depend on a good Math foundation.

**Empathy and Humor**.

A good math student will practice a lot, and make a lot of mistakes.

When I make a mistake, I just chuckle a little and correct it and go on.

If I make a big mistake I laugh out loud.

Life is funny if you approach it right.

In your life you will make a lot of mistakes.

It is up to you to decide how to deal with them.

This is very important if you want to maintain a good psychology.

I confess, I would not have earned a Ph.D. in Math IF I had not learned to laugh at myself and my mistakes.

Summary:

SPIKE Pedagogy which is necessary for a good Math Education.

The facts are that it is very difficult to deliver the SPIKE Pedagogy for each student in a group setting of many students which is how math is still being taught in many of our schools.

That is why Homeschool Math can be Superior to Public School Math.

In a typical classroom, the teacher will be going too fast for some students and they will fall behind and FAIL.

Any grade less than an A is essentially failure.

You either understand a Math Topic or Concept or Tool, or you don't.

And, the teacher will be going too slowly for some students and they will become bored and frustrated. Bad for them, too.

Grading on the Bell Curve is a HOAX.

Math performance is essentially bi-modal.

Either you understand a math topic, or you don't.

So, if a teacher tries to slow down as much as possible to keep fewer students from failing, this then makes the Math boring for the good students.

Boredom with a subject again creates bad psychology and ultimately, bad results.

Indeed, often the primary "motivation" for a good student in a math class is to just get a good grade by scoring high on the Bell Curve.

Conclusion:

Deliver Math Education to your students utilizing SPIKE Pedagogy.

See Special Offers below.

# Chapter 9: The SupraComputer

The SupraComputer was introduced to the world in about 2017 and will do things no ordinary computer will do.

It is based on the Raspberry Pi technology out of the United Kingdom.

It has a very secure Linux Operating System and is the size of a deck of cards, and thus, very easy to transport and totally secure since you can remove the SD card which contains all of your data.

The SupraComputer does many things for children and adults of all ages.

However, what makes it vastly superior to ordinary computers is that it comes loaded with Mathematica, Wolfram Language, and a pro version of Wolfram Alpha.

So, you can solve any Math Problem with Wolfram Alpha AND save it in a Notebook for future review. This you can't do with a regular computer or with the free online version.

Also, you can write incredible programs using the new Wolfram Language.

To fully understand it, simply go to:
https://supracomputer.org/

Once a student gets to Tier 4, the SupraComputer will be a great asset.

Not only can Wolfram Alpha solve the problems, it will now give the student a Step by Step explanation of how to do it manually, if the student wants to know.

This can be of great value if the student is taking a 20th Century Math Course where the student must master the old manual techniques.

# Chapter 10: STEM Math – The Full Story

This book is for Parents of a student who might be interested in some STEM subject like Science or Engineering.

This Chapter may be challenging to anyone who doesn't know much STEM Math.

However, I think you can probably understand the main ideas and themes even if you don't understand the math details.

If someone you know is a STEM person, you might want to have him or her read this Chapter and discuss it with you.

There are two approaches to learning Math: Heuristic and Rigorous.

A heuristic technique is an approach to problem solving, learning, or discovery that employs a practical method not guaranteed to be optimal or perfect, but sufficient for the immediate goals.

A rigorous technique is an approach which involves proving a meaningful statement is true with a series of logical deductions from a well-defined set of assumptions, called Axioms, Postulates, or previously proven Lemmas or Theorems.

Modern mathematicians and some math educators often try to teach math with a rigorous approach, which can be very tedious, boring and DIFFICULT to understand.

Difficult to teach too!

Classical mathematicians typically utilized a heuristic approach to learning and teaching and discovering mathematics, which can be MUCH easier and enlightening.

Heuristic is the approach I believe we should take when teaching Math at the High School level.

The wonderful 20th Century math teacher, Dr. George Simmons, agrees with this heuristic approach as demonstrated in his wonderful book, *PreCalculus Mathematics in a Nutshell*, which is the only textbook I utilize in Triad Math's Six Tier Program.

Any Parent or Teacher should at least read Dr. Simmon's Preface and each of the Introductions to his three Chapters on Geometry, Algebra, and Trigonometry.

It costs about $20 on Amazon.

Wonder why it isn't adopted by our high schools?

By the way, Dr. Simmons wrote what I consider the best Calculus and Differential Equations books before he wrote the PreCalculus book.

Dr. Simmons also wrote the best Topology and Modern Analysis book first, which is how I learned about him when I taught this subject as a professor.

This is the math like Hilbert Spaces used in the Math Models for theoretical physics subjects like Quantum Theory.

Dr. George Simmons was a Great Math Teacher.

So what should we be teaching our high school STEM students that will get them optimally prepared for a great 21st Century University STEM education?

**Numbers**: Real Numbers, HyperReal Numbers, and Complex Numbers.

We need to teach these number systems heuristically, not rigorously.

Real Numbers correspond to all of the points on a Straight Line and are expressed by the decimal number system.

Subsets of the Reals are Natural Numbers, Negative Numbers, Integers and Rational Numbers, all of which a STEM student must understand heuristically.

Non-repeating decimal Real numbers are called Irrational Numbers with Algebraic and Transcendental subsets. These are ONLY of interest to Theoretical Mathematicians and not needed for Science or Engineering.

Complex Numbers correspond to all of the points in a two dimensional Plane.

Complex Numbers are easy to understand heuristically and involve Trigonometry Functions and Infinite Series.

Euler's Complex Number Equation is the most important Math Equation for Science according to the great physicist Richard Feynman.

I agree, and it is Key to really understanding Complex Numbers. I teach this in Tier 4.

Complex Numbers are vital and necessary for many STEM subjects.

HyperReal Numbers include the Real Numbers and Infinitesimal Numbers. These are very important for a heuristic approach to Calculus.

Our ancestors approached Numbers Heuristically until the mid-19th Century.

Then they began to approach Numbers Rigorously with an Axiomatic Approach.

This is a much more difficult approach to numbers and probably should not be taught until a student decides to become a theoretical mathematician.

Indeed, there was a great split between Applied Mathematician, Scientists and Theoretical Mathematicians in the mid 1800's that was not resolved until 1966, coincidentally the year I earned my Ph.D. in Math.

Here's the story of what happened.

Mathematicians starting with the best ancient Greek Mathematician, Archimedes, used Infinitesimal numbers to solve many wonderful math problems such as relating the Area of a Circle to the Ratio of its Circumference to its Diameter, Pi = C/D = C/2R, Archimedes proved A = (D/2)x(C/2) = PixRxR.

Euclid had not done this in his famous Euclid Elements, the first attempt at rigorous math. (See Archimedes Tombstone Lesson in Tier 2.)

The creators of Calculus, like Leibniz (1600's) and Euler in the 1700's, utilized Infinitesimals in the wonderful creation of Calculus and Differential Equations.

BUT ALAS, when theoretical mathematicians began to make the Number Systems Rigorous in the mid 1800's, they could not figure out how to include Infinitesimal Numbers in their Axiomatic Number Systems.

SO, what did they do?

They BANNED THEM!

Wow. A bifurcation in STEM Mathematics.

Infinitesimal numbers make the understanding of Calculus heuristically much easier than a rigorous approach mathematicians adopted in the 1800's.

Applied Mathematicians and Scientists went their own way and continued to use Infinitesimal Numbers heuristically. They are so wonderful and useful!

Unfortunately, the theoretical mathematicians wrote our 20th Century Calculus books and went a rigorous and difficult way.

No more Infinitesimals!

Horrible setback for STEM Math Education.

Then guess what?

In 1966, Abraham Robinson figured out how to make Infinitesimals and HyperReal Numbers Rigorous and they are back in!

"Whoopie! Now STEM Math is easier again," so I thought in 1967.

Ooops!

Unfortunately, our Calculus books have not caught up yet.

Wonder why?

Could it be the huge investment in the current Calculus books?

But, of course, I use Infinitesimals extensively when I teach Calculus because they are a very good heuristic way to understand Calculus.

BUT, the Story gets much, much better in the 21st Century.

In 2009 to be exact.

So, just read on. The Math Education Plot thickens.

What are the ingredients of STEM Math?

Algebra, Geometry, Analytical Geometry, Trigonometry, Calculus, Differential Equations, all of which depend on . . . .

**Functions**. Polynomial, Trigonometric, Exponential and their Inverses.

**Infinite Series**. These are like infinite Polynomials.

**Math Tools** for analyzing Functions and Solving STEM Math Problems.

**Graphing a Function** to get a visual picture of its behavior.

**Calculus** which are the two Tools for analyzing the behavior of Functions, and understanding their Visual Representations called Graphs and the Area under their Graphs.

Differential and Integral Calculus and The Fundamental Theorem of Calculus are the reason we are here today.

**Differential Equations** which is how STEM Physical Math Models are created and how new Special Functions are discovered.

I realize that most people have difficulty understanding what this all means.

For what it's worth, many current High School teachers do too!

But, this is the Math a Student must learn to master STEM subjects.

Math is the Foundation of all STEM subjects, which essentially are based on Functions which are used to build STEM Models, which then are used to understand STEM topics and subjects.

Any Parent with a STEM student must deal with this situation, and now it is possible as you will learn!

**THE FUNDAMENTAL PROBLEM with the Standard Math Curriculum**

Wonderful Manual Tools were developed to deal with all of these Math topics by our ancestors who essentially only had Pencil and Paper type tools.

Unfortunately, these Manual Tools are very difficult to learn, master and use!

It's the main reason so many students develop an intense fear and dislike of math, and then many students do not pursue a STEM career.

Don't blame them. It was a great struggle for me too.

One has to deal with Numbers and arithmetic calculations.

Manual Algorithms were developed for addition, multiplication, division, and square roots of real numbers.

Time-consuming and error-prone. Hard to master.

And, then Manual algorithms for Complex Numbers. Ditto.

Logarithm, the inverse of the Exponential Function, became a wonderful tool for performing arithmetic.

Then the Slide Rule, or Slip Stick, based on Logarithms, became a wonderful tool used by all Scientists and Engineers for centuries.

Trigonometry Tables also were created and were indispensible in solving many math problems.

Calculus was invented in the 1600's which was the foundation of our modern Science and Technology world. Newton's Physics started it.

Calculus is used to analyze Functions.

Manual Tools were developed by Liebniz and Euler and others.

Wonderful Tools, but very difficult to learn and master and use.

Then, we had to learn to solve Differential Equations which often result in what are called Special Functions, i.e. Infinite Series, as solutions.

If you wanted to become a STEM professional, you had to first learn these Concepts and Manual Tools.

The Math Concepts are actually pretty easy to learn heuristically.

Unfortunately, All of these Manual Tools are very difficult to master and use. They create a Great Barrier for STEM students!

Until the 21st Century, or 2009 to be exact.

The Math Concepts are pretty easy to learn and understand.

Intelligence was necessary, but even more important was a lot of very hard dedicated work to master these Manual Tools.

Math was TOUGH and was often an insurmountable Barrier to STEM subjects.

**The Manual Tools, not the Concepts, were really the Barrier and Challenge.**

So what? Why am I telling you this?

Our Math Programs are still teaching these Manual Tools!

That's why!

Integral Calculus flunked many kids out of engineering schools since it was the first really difficult manual tool that had to be mastered.

It is called Techniques of Integration and involves finding Antiderivatives of Functions in order to apply the Fundamental Theorem of Calculus.

Much more difficult than the Manual Tools that led up to it.

Then Miracle #1 happened in 1972.

The first Scientific Calculator, the HP-35 was unleashed on the world.

Now Log and Trig Tables were obsolete.

The Slide Rule was obsolete.

Now, a STEM professional could do all arithmetic calculations very quickly and easily, AND the Scientific Calculator was an order of magnitude easier to learn and to use to solve math problems.

Needless to say this was a Crisis for Math Educators.

Many of their books and courses were obsolete!

Students rejoiced.

Many Teachers were angry and depressed.

Much of what they had been earning a very good living teaching was now in the dustbin of history.

They had to adapt to the new technology and reality.

Many textbooks were obsolete.

But, Calculus and Differential Equations, the workhorses of STEM, were not affected very much. Still the old Manual Techniques were all we had.

So for advanced STEM math, not much changed.

Until 2009! Miracle #2.

A massive Educational STEM Math Earthquake more significant than the 1972 Scientific Calculator Miracle #1 happened.

Some background first. A very interesting story. Then Miracle #2!

Computers were developed shortly after WWII and really took off in the 1950's.

By the 1970's, MIT developed a sophisticated Computer Algebra Program, Macsyma, which could solve Math Problems numerically.

A teenager in the UK, Stephen Wolfram, used Macsyma to solve physics problems, and soon went to the greatest physics graduate school in the world at CalTech and earned his Ph.D. in physics at age 20 in 1978.

Wolfram won the MacArthur Genius Award.

Wolfram then wanted to create a more powerful tool than Macsyma, and in 1988 introduced Mathematica to the world.

Mathematica was an incredible programming language unlike anything before.

Steve Jobs incorporated Mathematica into the Next Computer and this system was used to create the World Wide Web, by Tim Berners-Lee at CERN.

Okay! Miracle #2 which revolutionizes STEM Math Education.

Stephen Wolfram introduced an incredible Tool to the world in 2009:

**Wolfram Alpha**
https://www.wolframalpha.com/

Mathematica became more and more powerful, and after 30 years, Wolfram was able to create Wolfram Alpha.

Wolfram Alpha is a very sophisticated Mathematica Program that is hard to believe it is so powerful.

Guess what?

Wolfram Alpha will solve any Calculus or Differential Equation Problem immediately including problems that cannot be solved with the Classical Manual Techniques.

And, much, much more!

Truly amazing!

Solve STEM Math Problems in a minute or so that would take hours if you could do them at all with the Manual Tools still being taught.

**Wolfram Alpha is MUCH more revolutionary for STEM Math than the Scientific Calculator was four decades earlier in 1972.**

Now, starting in Middle School we can teach math to students that is virtually impossible to teach with the old, now obsolete Manual Tools.

Thus, I developed a new 21st Century Six Tier Math Curriculum utilizing this amazing Modern Tool to teach virtually all STEM Math starting in Tier 4 with Algebra and Geometry, like Conic Sections, as well as Trigonometry and Complex Numbers.

This continues in Tier 5, with Calculus, which can now be learned in about one semester, very easily.

Then in Tier 6, solving Differential Equations, quickly and easily in another semester.

Orders of magnitude easier than the old Manual Tools.

The Great Math Barrier of Integral Calculus and Differential Equations has been eliminated!

Now STEM Math is no longer an impediment to learning a Science or Engineering subject.

It's like a modern automobile replacing a horse and buggy for traveling.

A student can now learn all of the STEM math needed for Science and Engineering in a very small fraction of the time and effort needed for the Classical approach.

ALL high school textbooks are now obsolete.

All Calculus and Differential Equations books are obsolete.

Why?

They are loaded with all the very difficult Manual Tools which are now obsolete, and very difficult to learn and master and use.

No modern STEM professional uses them anymore.

Indeed, you don't really need a book to learn Calculus and Differential Equations.

Just some Notes, Exercises with Tutorial Videos and Wolfram Alpha.

All delivered with SPIKE Pedagogy.

Easy and Inexpensive!

Unfortunately, the Math Education System doesn't want to give up a multi-billion dollar set of products developed over the past 20th Century.

I can't blame them any more than I could blame the engineering professors who didn't want to abandon Slide Rules and Log and Trig Tables in 1972.

But, that's the world we live in.

It's just like Smart Phones replacing Landlines and many other classical obsolete technologies.

The Standard Math Curriculum is like teaching carpentry with the manual tools of the 1700 and 1800's.

Just visit a museum and imagine how difficult it was to use the manual tools and compare that to utilizing modern electrical tools.

Here's an analogy.

The Concept of “a hole in a piece of wood” is easy to learn.

How do you solve the problem of creating a hole in a piece of wood?

Without any tool it is probably impossible.

First, you could create a hole with a nail or spike and hammer.

Of course, it was difficult and you could split the wood.

Then, came a wonderful manual tool, the Brace and Bit.

This is how I was taught to create a hole when I was a boy in the 1940's.

It took a lot of effort to learn it and a lot of practice to master it, which I never really did.

It took a lot of slow, physical effort.

It was often impossible to drill a hole where I wanted one due to the limitations of this tool.

Then, came the electric drill.

Wow.

Much easier to learn and to use.

Much faster and less physical effort.

And, you could drill holes in more inaccessible places.

Wow! Brace and Bit obsolete.

Then, the electric drill with a battery. Even easier to use.

Analyzing a function with algebraic and manual graphing techniques is like making a hole with a hammer and nail.

Analyzing a function with the Manual Techniques of Calculus is like a Brace and Bit.

Hard to learn and difficult to perform. But, much better than a Hammer and Nail.

Analyzing a function with Wolfram Alpha is like a battery powered electric drill. Easy-peasy.

Now, any student can learn STEM math and apply it very quickly and easily.

Manual Calculus and Differential Equations techniques are OBSOLETE!

Math is NO LONGER a Barrier to learning a STEM subject.

That is why 21st Century Math Programs are vastly superior to 20th Century Math Programs.

That is why the Homeschool Math can be vastly Superior to Public School Math.

Eventually Public and Private School Math will catch up, I HOPE.

But, a multi-billion dollar industry will have to transform.

Math teachers will become Coaches.

Math will be delivered with SPIKE Pedagogy.

This book was created to give you more details about one Program that achieves these 21st Century Goals, Triad Math's Six Tier Math Program.

Furthermore, an amazing new Tool was invented in 2016, the SupraComputer.

This Tool makes STEM Math even easier to learn.

See Chapter 9.

Be sure to check the Special Offers too!

You have nothing to Lose, and MUCH to Gain!

# Chapter 11: Teacher & Coach

To learn Math, a Student needs both a Teacher and a Coach.

The Teacher selects the appropriate topics for a student and then explains each topic and gives the student exercises and feedback via a quiz.

The Coach monitors the student's activities and gives the student motivational feedback with both "carrots" and "sticks".

The "sticks" might be requiring the student to spend a certain amount of time studying math with the Teacher.

The "carrots" are giving the student positive feedback in the form of compliments and rewards for efforts and accomplishments.

Mistakes are unavoidable when learning math, just like any sport or skill.

The Coach should acknowledge the student's mistakes as a good effort and progress in learning the math concepts and skills.

DO NOT ever let a student feel s/he is a "failure" because of some mistake s/he makes. Celebrate mistakes as a sign of effort and progress.

A Coach should be able to come up with various "rewards" for a student's progress and efforts.

Recognition and sincere compliments are often the best rewards.

A Coach must be present in a student's life, and care about the student, and make persistent and consistent efforts to give the student these positive feedbacks. And, a kick when needed.

A Coach must be prepared to encourage the student to make efforts even when these efforts seem to not be producing good results.

All successful people go through periods of "doldrums."

Any successful person in the development of any skill will make many "mistakes". That's Life!

The Coach must be sure the student understands this and appreciates this.

Compare Math to some game, sport, music or any other skill the student likes. We all make mistakes.

The better we become the fewer mistakes we make, but we always make mistakes. The more we learn the more mistakes we will have made.

The Coach should try to help the student see where learning the concepts and skills of Math will help the student in other areas of interest to the student.

If a student is interested in any technical field s/he must realize the value of math in this field. The Coach should be sure the student is aware of this.

A Coach does not have to be the Teacher.

The Coach does not have to know much math.

The Coach will work with the Teacher.

It is possible that one person can be both the Coach and the Teacher but this is usually impossible.

I can be the Teacher thanks to modern technology.

You can be the Coach.

Or, perhaps you can find someone else locally to be your student's Coach.

Remember the Coach is not the Teacher, and does not have to know much Math.

# Chapter 12: How to Be a Great Coach

### Motivation and Learning Techniques.

### Intrinsic vs. Extrinsic

**Intrinsic Motivation** . . . is when a student studies and learns math simply for the internal satisfaction and enjoyment.

It is what motivates any person to play a game or sport.

Once a student starts to learn math and gain confidence and self-esteem, Intrinsic Motivation often sets in.

This is what we want as a Coach and Teacher.

However, for many students this takes some time.

In the meantime, a Coach can use Extrinsic motivators.

**Extrinsic Motivation** . . . is when a student wants something that the study of math will provide. That "something" is an extrinsic motivator.

For example, if a student wants to study any STEM subject then that is an extrinsic motivator since Math is necessary for virtually any STEM subject.

If a student wants to enter any technical field in industry or the military, then Practical Math is necessary and that is an Extrinsic motivator.

Triad Math's Tiers 1 and 2 provide this necessary math and also are a great foundation for future math studies.

Ironically, the Standard Math Curriculum (SMC) IS NOT necessary for this goal.

If a student wants to excel on the SAT, then Triad Math's Tier 3 program will satisfy this need.

Again, this is quite different, and easier, than the Standard Math Curriculum.

Of course other extrinsic motivators can include any type of rewards.

This can include grades, praise, privileges, money or many other things the student might want including the avoidance of some type of punishment.

However, I find that when the Proper Content and SPIKE Pedagogy are utilized by the Teacher, then usually the student becomes intrinsically motivated since most of us enjoy anything that is challenging and we succeed at.

Let's assume that your student is motivated enough to study math.

It is imperative that the math be taught in such a way that the student is successful. Only that will lead to intrinsic motivation.

Learning Techniques.

To successfully learn math, a Coach should teach the student to engage in certain Learning Techniques.

This is different from teaching the math itself.

That is why a Coach and Teacher are two different necessary components of a good math learning experience.

I recommend Dr. Barbara Oakley's great book, "*A Mind for Numbers – How to Excel at Math and Science, even if you Flunked Algebra*", for an elaboration on what I am going to recommend to you as a Coach and to any student.

First, be sure the student studies topics in a proper sequence so that the student always has the necessary prerequisite knowledge for the topic at hand.

Go back and fill in any deficiencies you can identify.

Simply follow the Six Tiers if you utilize me as your Teacher.

If a student already knows a Topic, it will simply be a quick review.

Second, be sure the student studies a new topic with a Focused approach by studying the Notes with the Tutorial Video and then working on the exercises.

Do this for a reasonable length of time until either the student understands the topic and how to do the exercises, OR gets confused and tired.

Then STOP. Take a break!

Third, have the student engage in various activities so his or her mind can go into an Unfocused Mode.

This is when the subconscious mind processes the focused activities.

We don't understand how this really works, but it often does.

Usually this involves some routine task or habit that requires little thought.

Do some routine boring chores.

Watching entertaining videos or playing games may not work since they distract the subconscious mind.

Who knows?

Try various things and see what works for the student.

Fourth, have the student then engage in another Focused session and work on the same topic.

Now the topic seems more understandable and the confusion lifts some.

It's pretty amazing how often this works.

Persistence will yield success.

No one to my knowledge knows how this really works, but it does often work.

Many great thinkers have confessed this is how they often solve a problem or discover a new concept.

It certainly works for me and I use it all the time.

Try it. Give it a chance to work.

It is important for a Coach to explain and convince a student that s/he will succeed if s/he practices and tries hard enough and perseveres.

I can remember many times when, as a student, I struggled with a new Concept and then after several Focused and Unfocused Sessions it finally fell into place.

That's how I created my Thesis for my Ph.D. in Math.

Sometimes, after I mastered a Topic, I wondered why I ever had a problem with it in the first place.

Often, you have to try many things and go down many blind alleys before you achieve the understanding and solve the problem.

Failures then lead to Successes.

The more you believe in your capabilities the more success you will have.

Yes, you will have failures and frustrations.

The more difficult the problem or concept the more you will experience this.

But, the greater the reward and satisfaction will be when you achieve the breakthrough and achieve success.

A good Coach will explain these things and encourage the student.

Celebrate Mistakes as Progress too.

Success does breed Confidence and more Success.

But, Failure should also be a sign of Progress in ultimately achieving Success.

Success is built on the back of many Failures.

A student must “learn to learn” and a Coach can greatly facilitate this.

The Coach and Teacher must work in tandem.

# Free Resources

Simply go to www.CraigHane.com to get the current free resources available from Dr. Del.

The Video Library Tab will yield many videos Dr. Del has created, and is an ever expanding Library.

These videos are all YouTube videos.

Some are also YouTube videos of others that Dr. Del finds inspirational and informative.

# Special Offers

Go to www.TriadMathInc.com/SO for the current Special Offers.

Dr. Del and Triad Math, Inc. like to give students and families actual training so they can evaluate the methods our Programs use to determine if they would then benefit from some of our training products.

Seeing is believing.

Your experience is the only one that counts for you.

So, go take advantage of one of our current Special Offers.

Dr. Del wants the best for you and your family.

You have Nothing to Lose, and Much to Gain!

# Who is Dr. Del?

Dr. Del is a pen name used by Delbert Craig Hane, Ph.D.

You may get a full biography at:
https://craighane.com/vita-of-craig-hane-ph-d/

or just go to www.craighane.com and press the Vita Tab.

In a nutshell, Dr. Del has been a Learner of Math and a Math Teacher for over 75 years.

75 years?

Yes. Craig was taught the decimal number system and how to count and add by his Uncle Jack (Davis) who was a barber and builder, using Cheerios in about 1942-3 when Craig was five years old.

This was the beginning of WWII for the USA and most parents were heavily involved and had little time to teach their children at home.

When Craig was five years old, he was enrolled in the first grade at Putnamville, Indiana, a four room school with eight grades.

He was the youngest student in the first grade with teacher Miss Lewis.

He also was the only student in this first grade who had been taught to count in the decimal number system and

Miss Lewis had him help her teach his classmates to count.

So, Craig had his first teaching experience at age five.

No doubt this was a life transforming experience.

You are urged to have your children teach other children things they have learned as soon as possible.

Of course, Craig went on to learn Math from other teachers and teach other students Math the rest of his life right up to the first writing of this book in July, 2021.

Learning and Teaching Math are just Two sides of the Same Coin.

Also, Craig applied Math to many businesses and adventures too!

Applying Math to all sorts of situations in the real world can lead to all sorts of wonderful successes.